More than ten years after NCTM’s *Curriculum and Evaluation Standards for School Mathematics* (1989) first called for reforms in mathematics education, many schools still have not implemented those reforms. Where the Standards did take hold, they often failed to survive. As *Principles and Standards for School Mathematics* (NCTM 2000) notes, “Despite the concerted effort of many classroom teachers, administrators, teacher-leaders, curriculum developers, teacher educators, mathematicians, and policymakers, the portrayal of mathematics teaching and learning in Principles and Standards is not the reality in the vast majority of classrooms, schools, and districts” (p. 5).

This article looks at the past reform movement in mathematics from the perspective of a classroom teacher who has successfully refocused the classroom curriculum around problem solving, reasoning and proof, communication, connections, and representation. Although this article primarily will examine only one component of the reform agenda, this component is central to the reform movement. It involves moving mathematics instruction away from teaching mathematics through drill and practice and toward teaching mathematics through problem solving.

I base my comments on my own classroom experiences and the experiences of other teachers who have found that problem solving can be difficult to teach and even more difficult for students to learn. First, I examine why implementing the past NCTM Standards in their classrooms was so difficult for some teachers. Second, I offer suggestions to help teachers implement the revised Standards.

**What Went Wrong**

When faced with the expectation that they teach problem solving, many teachers unfortunately engage in what I call the “blame game.” This reaction can include four stages and consists of searching for someone or something to blame for the difficulties that teachers encounter when attempting problem solving with children. Teachers must be aware of these potential challenges because sometimes knowing what to avoid is as important as knowing what to embrace.

**Stage one of the blame game**

The first stage involves blaming yourself. Many teachers become convinced that their lack of knowledge is the source of their difficulties. Typical comments that teachers make at this stage include the following:

- “I could never figure out story problems when I was in school; how can I teach students how to do them?”
- “I’ve tried to teach problem solving, but it seems like I always end up showing the children what to do and it turns into just another type of drill-and-practice exercise.”

Teachers often try to expand their knowledge by taking college classes or attending workshops on...
Figure 1

Children solving problems in ways that make sense to them

The problem:
Hillary is 52 inches tall and Chloe is 39 inches tall. How much taller is Hillary than Chloe?

Below are typical children’s solutions. Although some of the strategies may seem unorthodox and inefficient, they represent children’s genuine attempts to make sense of the problem and communicate their solutions using a drawing, a manipulative, an oral description, or a computation procedure.

Jonathan (first grade):
Jonathan first drew 41 lines to represent Chloe’s height in inches. He then recounted the lines and discovered that he had drawn 41 instead of 39, so he crossed out 2 lines. Next, he drew 52 lines to represent Hillary’s height. He then proceeded to cross out one line at a time on Chloe’s row and on Hillary’s row. He continued this process until he had crossed out all the lines on Chloe’s row. He then counted the remaining lines on Hillary’s row and wrote 13 for his answer.

James (first grade):
James used a popular classroom manipulative. He made a row of 39 tiles on the floor “cause they wouldn’t fit on my desk.” Then he made a row of 52 tiles and counted “the ones that were more ‘cause that’s how much more she is.”

Jose (second grade):
Jose used a different manipulative than did James—his fingers. Jose put 39 in his head and then used his fingers to count on from 39 to 52. He paused as he reached 49 so he could put “10 in my head for later.” When he reached 52, he held up 13 fingers, so he said, “She is 13 inches more big.”

Natasha (second grade):
Natasha used an approach similar to the direct modeling method that Jonathan used, but she has less of a chance of making a counting error because she used bars to represent the quantity 10. Also, her approach might be more accurate than the counting-on method used by Jose because she does not have to keep track of quantities in her head.

Brandi (third grade):
“52 is 40 + 12 and 39 is 40 – 1 so it’s 12 up and 1 down and that makes 13.”

Brandi used an approach common to some children: a familiar reference point from which to work forward and backward in order to facilitate her computations. In this case, she chose to use a multiple of 10 so she could “solve the problem in my head.”

Eli (third grade):
“52 is 4 inches more than 4 feet and 39 is 3 inches more than 3 feet, so minus them and you get 1 inch and 1 foot and that’s 13 inches.”

how to teach children problem-solving heuristics (look at the problem, choose a strategy, solve the problem, look back) or problem-solving strategies (guess and check, make a model, look for patterns, estimate, and so on). Although this type of professional development can be helpful, good teachers
of problem solving do not necessarily need to be expert problem solvers themselves. Instead of teaching children how to solve certain types of problems using traditional problem-solving strategies, teachers can encourage children to solve problems in ways that make sense to them (see fig. 1). Engaging in sense making while solving problems is one way that children deepen their understanding of mathematics.

Once children have solved problems, they can share their personal strategies with classmates and respond to questions or comments that their peers and teacher make. Putting their solution processes into words is a second way that children deepen their understanding of mathematics. When children write or talk about how they solved a problem, they must organize and clarify their thinking as they attempt to communicate their ideas clearly and completely. Finally, children can compare all the different solutions that their classmates have shared and examine how they are the same or different. Reflecting on how different ways of solving problems are related or connected is a third way children deepen their understanding of mathematics.

Stage two of the blame game
The second stage in the blame game involves blaming the mathematics problems. Typical comments that teachers make at this stage include the following:

• “If someone would just write some good problems, it wouldn’t be so bad.”
• “Sometimes the problems are so wordy, it’s more like a reading test than a mathematics activity.”

Teachers at this stage frequently ask, “Who has written the book of good problems, and where can I buy a copy?” However, there usually is nothing wrong with the problems that teachers already use. Although using “good” problems is important, the quality of the problems does not necessarily determine the success or failure of a problem-solving program. In fact, some of the best problems for children to solve exist in the day-to-day activities that occur in the classroom. To take advantage of these problems, teachers must listen to what children say and do, and then help them become aware of the mathematics embedded in these events (see fig. 2).

Teachers can construct problems out of the mathematical data that surrounds children as they participate in lunch count, share classroom and playground equipment, construct objects with blocks, create designs with pattern blocks, or explore Internet sites.

Stage three of the blame game
The third stage of the blame game involves blaming the students. Out of desperation, teachers make comments such as the following:
• “My students just don’t try. Unless I tell them what to do and how to do it, they just sit and wait for me to do it for them.”
• “Children are so impatient. Unless the problem can be solved in five seconds or less, they don’t want to have anything to do with it.”

Fortunately, their students’ lack of particular skills or abilities is not the reason that most teachers have so much difficulty teaching problem solving. Children appear to be natural problem solvers, and problem solving is one of the primary functions of the human brain (Jensen 1998; Sylwester 1995). Although children frequently solve problems in unusual or confusing ways, their solutions often reflect the ingenuity and creativity of young minds (see fig. 3). When children are given the chance to solve problems in ways that make sense to them, they are capable of solving problems often considered beyond their abilities by using their informal mathematical knowledge to—

• take ownership of the problem and craft a solution from a personal perspective;
• develop unique strategies for solving a wide range of problems;
• construct meaningful connections between what they know and what they are learning; and
• grow as capable problem solvers who can solve problems with confidence, communicate their strategies orally and in writing, and reflect on the solutions of others.

Children can not only solve problems but also write their own problems (see fig. 4). Students then solve one another’s problems and the authors of the problems agree or disagree with each solution. These problems serve a dual purpose: They give children a chance to take control of the mathematics curriculum and they give teachers a way to examine what children think are the characteristics of good problems.

Stage four of the blame game
The final stage of the blame game becomes the turning point for many teachers. Either they pass through this stage and go on to become successful teachers of problem solving or, after blaming everyone and everything, they simply give up and leave this difficult task to someone else. In this final stage, teachers blame “those other people” and make comments that include the following:

• “If the district curriculum specialists would leave us alone and let us teach mathematics, we wouldn’t have all these problems.”
• “How can I be expected to teach problem solving along with all the other things I am supposed to cover in the state-mandated mathematics curriculum?”

The reason that many teachers feel frustrated in their attempt to teach problem solving, however, is not that individuals outside the classroom are imposing unreasonable expectations on them and their students. In fact, the world outside the classroom is full of problems that need to be solved. Many state departments of education support the teaching of problem solving and include these types of questions on state tests (see fig. 5).

If teachers are not the problem, and the mathematics problems are not the problem, and the students are not the problem, and “those other people” are not the problem, why is problem solving so difficult to teach and even more difficult for students to learn? A very simple answer to this question exists: Teaching problem solving using the traditional, drill-and-practice model of mathematics instruction is difficult. Although this type of instructional model is effective in helping children remember mathematics facts and computation procedures, it is less effective in helping children learn how to become problem solvers.

Problem solving requires understanding that goes beyond memorized facts and computation algorithms. Most teachers who work with young children eventually realize that although they can transfer information to young learners, transferring understanding is very difficult. This is because each individual crafts his or her understanding as information is transformed into useful and usable knowledge. Problem solving requires the ability to use information purposefully, not just the ability to memorize and remember it.

The Challenges That Teachers of Problem Solving Face
Teaching problem solving poses many challenges for teachers. First, teachers must change not only what they teach but also how they teach. These changes in methodology must extend beyond fine-tuning existing instructional practices; teachers must adopt fundamentally different ways of
teaching and assessing children. In addition, teachers must acquire subject-matter knowledge that is deeper and more flexible than that required to follow a lesson plan or directions in a teacher’s manual. As teachers of mathematics facts and formulas, teachers must transfer what they know to their students. As teachers of problem solving, however, their role shifts to helping students extend and expand what they already know in order to learn even more. Teachers can do this by using children’s original solutions to problems as a basis for classroom conversations and then ensuring that the students discuss significant mathematics.

A second challenge is that teachers must model for students problem-solving abilities that they neither possess nor have seen demonstrated by others. In short, the NCTM Standards ask teachers to be teachers that their professional development has not prepared them to be and that they have not experienced in their own lives as learners. One way that teachers can learn to become successful teachers of problem solving (and students can learn to become problem solvers) is to spend time in the company of others who can—

• model appropriate behaviors, dispositions, and skills;

• engage the novice in an ongoing conversation within a community of learners; and

• give the novice timely feedback.

Unfortunately, these models and learning communities do not exist in many of the universities where teachers are trained or in the schools where they teach. The good news is that these models are becoming available as more teachers incorporate problem solving into the curriculum.

A third challenge for teachers of problem solving is that they may need to change some of their most basic beliefs about what constitutes mathematical literacy. At the heart of these beliefs is the very definition of mathematical literacy:

• Do children demonstrate mathematical literacy when they memorize facts and computation algorithms? Is problem solving the ability to identify which computation procedure will find the answer to story problems?

• Or do children demonstrate mathematical literacy by using mathematical information purposefully to build relationships between the big ideas in mathematics? Is problem solving the ability to solve a wide range of problems even when the solution to the problem is not immediately known?

Figure 3

Impossible problems

Problem solving often requires creative as well as critical thinking. In order to stretch children’s thinking, I sometimes pose problems that encourage children to “play with ideas” and “think outside the box.” These problems have earned the nickname “impossible problems” because they appear to lack a mathematical solution.

How can two children share 5 pennies?

Zachary: “You could cut one of the pennies in half and they would each get 2 1/2 cents, but if you cut money in half, it isn’t worth anything. So I don’t think you can do this problem.”

Chloe: “They could each get 2 so it would be fair, and put the other penny in the bank until they got another 5 cents, and then take the penny out and they each get 3 more cents.”

Ariel: “They could buy a candy bar that costs 5 cents and break it in half, and then they each get half a candy bar and it’s a fair share.”

Eli: “They could buy a lottery ticket and when they win they could split the $1,000 prize, and they would each get $500 and they would be rich forever.”

How can three children share 10 balloons?

Benjamin: “Well, it’s impossible because if you try to cut a balloon in half it will pop.”

Andrea: “They each take 3 and give the other balloon to their teacher because she needs to be happy too.”

Austin: “When no one is looking, let one of the balloons go and say, ‘Oops, it just slipped,’ and then share the others so they each get 3 balloons.”

Timothy: “Sell the balloons for 30 cents and share the money so they each get 10 cents.”
The fourth, and perhaps most difficult, challenge is that teachers may need to convince parents and others that the changes that the NCTM Standards call for are necessary and will significantly raise student performance.

**Conclusion**

In order to realize the vision of the NCTM Standards for mathematics instruction, teachers need support and encouragement. Both students and teachers must learn new roles, behaviors, and

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**Figure 4**

Student-authored problems with their classmates’ solutions

**Brandi’s problem:** My Mom baked 4 dozen cookies for the bake sale. If there are 6 cookies in each bag and if one person bought 3 bags for $3.00, how many dozen cookies were left?

<table>
<thead>
<tr>
<th>Austin’s Solution</th>
<th>Andrea’s Solution</th>
<th>Todd’s Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>12+12+12=48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12+12=24+6=30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1 2 ½</td>
<td>1 ½</td>
<td></td>
</tr>
<tr>
<td>2 ½ dozen</td>
<td>2 ½ dozen</td>
<td></td>
</tr>
</tbody>
</table>

**Zachery’s problem:** When we went out for pizza, my dad ordered 2 pizzas. My parents ate 3 pieces, each of them, and my brother and sister and I each ate 2 pieces. How many pieces of pizza were left? This is how they cut them.

<table>
<thead>
<tr>
<th>Hillary’s Solution</th>
<th>Brandon’s Solution</th>
<th>Chloe’s Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 pieces</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td></td>
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<tr>
<td></td>
<td>¼ left</td>
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</tbody>
</table>
skills. Learning to solve problems and learning to memorize information require profoundly different ways of thinking and behaving. Instead of embedding the teaching of traditional mathematics skills in the context of problems, many teachers will continue to try to teach the traditional content through drill and practice along with a separate problem-solving curriculum and find the task to be overwhelming.

Teachers need time to learn new content, time to learn old content in greater depth, and time to construct personal understanding. Simply learning new mathematics content is not enough; teachers must also learn how to present the content so that children learn mathematics with understanding. Teachers must start asking the right questions because the questions they ask will determine the answers they uncover. Instead of asking, “What is the most efficient way to teach mathematics content?” teachers should ask, “What is the most effective way for children to learn this content?” Although these two questions may seem similar, they yield surprisingly different answers. The first question focuses on the quantity of information to teach and the second focuses on the quality of the learning process.

Teachers also need professional development that includes continuing education at the university level, as well as the opportunity to attend conferences and workshops that introduce them to different ways of teaching mathematics. Based on my experiences as a workshop presenter, however, I realize that the traditional one-shot workshop will not result in the desired changes. Teachers must visit classrooms where problem solving is being taught successfully. Teachers need to see for themselves that the problem-solving approach will work in another ordinary classroom before they believe that it can work in their own classrooms. Conversely, many teachers cannot think about how to implement a problem-solving approach in their classrooms until they believe that it is actually possible.

Finally, teachers need a class size that is appropriate to facilitate student understanding of mathematics. If we define learning in terms of memorizing facts and algorithms through drill and practice, class size is somewhat irrelevant; thirty children can sit at their desks and complete arithmetic worksheets as easily as ten students. If, however, we define learning in terms of using mathematics in purposeful ways, class size is very relevant and the great number of students in many classrooms severely limits the construction of understanding by all children.

**Final Thought**

In most school districts, the responsibility for organizing, budgeting, and scheduling staff development belongs to the administrators of the district. Many administrators, however, lack an understanding of NCTM’s Standards and the type of staff development activities that would most benefit teachers. Therefore, these administrators may need to step out of their traditional oversight role and participate more directly in the staff development activities of their schools or districts. Because NCTM’s Standards call for an entirely new way of teaching and learning mathematics, administrators may need to move away from administering staff development activities and toward leading their staffs with insight. Teachers may need to do the following:

- They should encourage school and district administrators to attend training sessions on problem solving, visit classrooms where problem solving is being taught successfully, and join with teachers to build a shared vision of mathematics instruction.

### Figure 5

**Problem-solving questions similar to those on a state test for third grade**

1. Mr. B. lit a brand-new candle. After three hours, the candle had one-fourth of the wax remaining. How long will it take for the candle to burn completely?

2. Jason bought these things. If he gave the clerk a five dollar bill, what coins and bills will the clerk probably use to give Jason his change?

| 2 note pads: | $0.35 each |
| 1 pen:      | $0.79 each |
| 1 ruler:    | $1.25 each |

3. Which game did the Braves lose by the most points?

<table>
<thead>
<tr>
<th>Day</th>
<th>Braves</th>
<th>Mariners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuesday</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Friday</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Saturday</td>
<td>2</td>
<td>3</td>
</tr>
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</table>
• They should ensure that school and district administrators schedule staff development activities over an extended period of time because most teachers will need several years to become confident and competent teachers of problem solving. Teachers will need ongoing support and training to overcome the difficulties that they will face in their classrooms.

• They should help school and district administrators become knowledgeable leaders who can build local consensus in the school, district, and community about NCTM’s Standards and a problem-solving approach to learning mathematics. Any deviation from the traditional approach to teaching mathematics frequently produces a strong backlash from some school board members, parents, or politicians. Teachers and administrators must be prepared to address the concerns of the critics.

If teachers and administrators fail to fully understand the magnitude of the changes that NCTM’s Standards call for, reaching consensus in any school or district about what students should know and be able to do as mathematicians will be difficult. Some high school and middle school teachers will continue to blame teachers of the grade levels below theirs for not adequately preparing students for the challenges of their classrooms. Similarly, some elementary school teachers will continue to blame parents for not preparing children for school and some parents will continue to blame schools for not teaching children mathematics the way that they were taught. The blame game must stop. If educators cannot reach a consensus, the “Math Wars” that have been fought in California, Oregon, and elsewhere will continue to weaken the reform movement well into the future.

References